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ABSTRACT

The problem investigated was the analysis of a complex figure by identifying simpler figures embedded in it. The primary goal was to determine the level of sophistication of analysis employed by children in the primary grades; a secondary goal was to determine if students could be led to expand their analyses. Drawings and problems were prepared by the experimenter. Fifteen students were randomly selected from seven-to-ten year old summer school students; average age was 8.1 years. Each student was interviewed individually. Results showed that subjects almost universally failed to identify overlapping figures, that the subjects seemed to employ search techniques and that these techniques were most frequently used by older students, that there was a lack of statistical relationship between age and problem solving ability, and that limited instruction was somewhat effective. (DT)

GEOMETRIC PROBLEM SOLVING ABILITIES OF CHILDREN IN THE PRIMARY GRADES

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The purpose of this study was to extend results obtained by Yakimanskaya (1958, 1959, 1962) in the U.S.S.R. and recently collected in translation by Kilpatrick and Wirszup (1970, 1971). The particular problem investigated was the analysis of a complex figure by identification of simpler figures embedded in it.

Yakimanskaya (1958) described this problem as "the ability to read a diagram completely" and stated that this ability depends on "the various possible interrelationships between sensory and conceptual abstraction [p. 110]." In particular he was concerned with both quantitative analysis (number of embedded figures identified) and qualitative analysis (whether disjoint or overlapping figures were identified and the order of identification) employed by his subjects.

As a result of 700 experiments with 350 students in grades four through eight, Yakimanskaya concluded that the method of analysis of a figure seemed to reflect the level of sophistication of a subject's analysis, synthesis, and abstraction processes; and he identified two such methods. One seemed to be characterized by the "cutting away from the whole figure a single closed part [p. 112]." The order of identification of embedded figures seemed arbitrary, and the number of figures identified in this way was small relative to the total number of embedded figures. This method displayed "the weakness of sensory abstraction, which does not incorporate the processes of analysis and synthesis [p. 114]."

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A second method seemed to be characterized by the identification of related figures. The manner of relating was either sensory (e.g., all figures with a common side) or conceptual (e.g., all triangles, then all trapezoids, then all rectangles, and so on). Within each conceptual category, identification might proceed on the basis of a common base or a common angle.

Even by the second method, identification of overlapping figures seemed difficult. Further instructions from the interviewer increased the number of such figures identified, but Yakimanskaya concluded that "failure to identify overlapping figures is caused ... by a poorly developed abstraction process [p. 115]."

Yakimanskaya (1959) grouped seventh grade subjects into two categories on the basis of speed and accuracy in solving a series of similar geometric problems. After working these problems, the subjects were asked to identify two overlapping triangles embedded in complex figures. Those subjects who identified the essential information in the initial geometric problems isolated the overlapping triangles and explained their success by pointing out the common side of the triangles. Those subjects who were not able to identify the essential information in the initial geometric problems were not able to isolate the overlapping triangles. These subjects seemed to rely on visual characteristics of the figures rather than relational characteristics.

Yakimanskaya (1962) pointed to the need to incorporate into the elementary school curriculum geometry problems which foster the development of the methods of analysis typically used by students who were successful at focusing on the essential characteristics of a problem. He illustrated this suggestion with a list of appropriate problems (pp 158-159). Semušin (1970) alluded to similar problems (p 80).

Wirzup (1971), in reporting the five developmental levels of the 1968 Soviet geometry program, indicated that in part these levels are sensitive to the growth of the ability to isolate and compare portions of a single figure or of related figures. The growth of this ability can be considered a generalization of Yakimanskaya's results.

STATEMENT OF THE PROBLEM AND HYPOTHESES

The primary goal of the present study was to determine the level of sophistication of anaylsis of geometric problems which children in the primary grades employ. A secondary goal was to determine if the same children could be led to expand their analyses of complex geometric figures. In particular, the following hypotheses were developed:

1. Among students in the primary grades, overlapping and non-overlapping figures are identified equally often in the context of complex drawings.
2. Among students in the primary grades, the order of identification of figures is random.
3. Among students in the primary grades, geometric problem solving ability is not related either to age, sex, or the ability to identify figures.

Operationally for this study, a figure was classified as overlapping if its interior was cut by any line segment in the given drawing. A figure was classified as non-overlapping if the intersection of its interior and every line segment in the given drawing was the empty set.

Geometric problem solving ability was measured by a series of three problems which are described in the next section.

METHOD

Materials: The experimenter prepared drawings and problems for use in the interviews of subjects. One of the problems suggested by Yakimanskaya (1962) was used intact. The remaining drawings and problems were designed to help identify problem solving skills. The drawings are included as Appendix A.

The materials were developed during two pilot studies. The first of these was a very informal trial with a class of second grade students in DeKalb, Illinois. The primary purpose of this trial was to acquaint the experimenter with language appropriate to students of the primary grades. Only anecdotal records were kept.

The second pilot study was conducted two days prior to and in the same school as the study being reported. The purpose of the second pilot was to check the suitability of the interview protocols, the data recording procedures, and the geometric problems. Complete data were kept on the subjects of this pilot study. Some modifications in the protocols were made during the pilot study, with one segment eliminated.

Subjects: The subjects (N = 15) for the study were randomly selected seven- to ten-year old summer school students enrolled in an ungraded art enrichment class in a suburban elementary school. The average age was 8.1 years; 12 of the subjects were female. At the request of the principal of the school, one parent of each subject was phoned by the experimenter to obtain permission for that child to participate in the study All parents gave their permission. The study was conducted in July 1972.

Procedures: Each subject was interviewed individually by the experimenter. The interviews were conducted in an interior room normally used by the school for individual testing. The order of protocols were as follows:

1) An initial, brief period was used to learn the subject's name and to acquaint him with the presence of the experimenter and the environment for the interview. The subject's age was determined.

2) The subject was asked if he knew what a triangle was. He was given a blank piece of paper and a pen and was asked to draw a triangle He was then asked to draw a different triangle. Explanation of the word "different" was neither offered nor given if asked for. The drawings were then removed.

3) The subject was presented Figure I-1. (See Appendix A) Which has already been drawn on a piece of newsprint, and was asked if the figure was a triangle. (All subjects agreed that it was.)

4) The subject was presented sequentially with Figures I-2. through I-5, each drawn on a separate sheet of newsprint. For each figure he was asked to trace with his finger each triangle in the figure. The experimenter recorded the order in which the triangles were traced by referring to a master diagram (out of sight of the subject) whose vertices were labeled. After the subject had finished analyzing each figure, he was asked if there were any other triangles. Each subject stated that there were no more.

5) The subject was asked to watch while the experimenter traced around one of the overlapping triangles (triangle ABE in Figure I-5a)

embedded in Figure I-5. The subjects were asked to name the figure thus traced. Upon request, the experimenter traced around the figure again. If the subject correctly identified the figure, he was asked if there were other triangles in Figure I-5 not previously identified. His responses were recorded. Figure I-4 was presented again, and the subject was asked to find any triangles not previously identified.

6) Figure II-1, which was reproduced on ditto paper, was presented to the subject, and he was asked to draw one line to form two triangles (Problem 1). After he finished, the drawing was removed and Figure II-2 was presented. The subject was asked to draw two lines to form three triangles (Problem 2). After he finished, Figure II-3 was presented. The subject was asked to draw three lines to form four triangles (Problem 3).

All hypotheses were tested at the .01 level.

RESULTS

Hypothesis 1 was tested by comparing the total numbers of non-overlapping and overlapping figures identified by the subjects against the total numbers of such identifications possible. These data are presented in Table 1.

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INSERT TABLE 1 ABOUT HERE

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The distributions were compared by computing χ^2 statistics, df = 1. The three χ^2 values for the data from Figures I-2, I-4, and I-5 were 15, 43, and 58, respectively. All were significant at the .01 level.

Hypothesis 2 was tested by comparing the order in which subjects identified figures with all possible orders in which the figures could have been identified. Since so few overlapping figures were identified, the analyses were conducted only for non-overlapping figures. The distributions of the subjects' responses in analyzing Figures 1-2, 1-3, 1-4, and 1-5 are presented in Table 2.

INSERT TABLE 2 ABOUT HERE

For each of the Figures 1-2 and 1-3, the total numbers of subjects exhibiting each order of identification were compared with the random expectation of 7.5 in each category. The computed χ^2 values, each less than 1.0, df = 1, were non-significant. The distributions of subjects by age were compared by applying the Kolmogorov-Smirnov approximation to χ^2 (Siegel, 1956, p. 134). Neither computed χ^2 value was significant.

For Figure 1-4, one would expect that if responses were given in random order, then $\frac{1}{3}$ of the subjects would identify the non-overlapping triangles in either left-to-right or right-to-left sequence and that $\frac{2}{3}$ of the subjects would employ some other order. The observed distribution of subjects was compared with the expected values of 5 and 10, respectively. The computed χ^2 value, 19.2, df = 1, was significant at the .01 level. The distributions of subjects by age according to left-to-right and right-to-left identification were compared and found to be not statistically different.

For Figure 1-5, one would expect that if the responses were given in random order, then $\frac{1}{3}$ of the subjects would identify the non-overlapping triangles by using a circular order of identification. The comparison of observed and expected distributions yielded a χ^2 value of 11.1, df = 1, which was significant at the .01 level. The distributions of subjects by age according to clockwise and counterclockwise identification were compared and found to be not statisticallly different.

For the following reasons, only part of hypothesis 3 could be tested: 1) The sample, though randomly selected, was predominantly female. 2) So few subjects identified overlapping figures that there was no basis for categorization on this variable. The sole analysis completed was to determine whether there was a relationship between problem solving ability and age. These data are presented in Table 3.

INSERT TABLE 3 ABOUT HERE

The distributions were compared by application of the Kolmogorov-Smirnov test..So few subjects solved problems 2 and 3 that no analysis was completed.

One further analysis was completed. During each interview, one overlapping triangle in Figure 1-5 was pointed out to each subject. The subject was then given an opportunity to identify other overlapping triangles in Figures 1-5 and 1-4. Eleven subjects identified one overlapping triangle not previously recognized in Figure 1-5. In each case the particular triangle identified was the triangle whose interior was the relative complement (relative to the interior of the quadrilateral) of the interior of the triangle pointed out by the experimenter. Of these eleven subjects, six were able to identify at least one overlapping triangle in Figure 1-4. The distributions of subjects is presented in Table 4.

INSERT TABLE 4 ABOUT HERE

DISCUSSION

A number of limitations of this study need to be kept in mind in the ensuing discussion. First, the small sample size precluded the detection of subtle differences, if any actually existed. Second, the sex bias of the sample prevented wide generalization. Third, the methods of quantification limited the kinds of analysis that could be performed. Fourth, the nature of atypicalness of these summer school students, if any, was not known.

In spite of these limitations, several conclusions seem to be warranted. First, the subjects almost universally failed to identify overlapping figures. Possibly, this was due to a lack of attention to geometry in the curriculum of the primary grades. On the other hand, if a child's perceptual ability develops in a quasi-Piagetian pattern, then the observed result may have reflected inadequately developed perception. In either case, instruction might have increased the number of identifications of overlapping figures. Another possibility is that the subjects may not have understood the directions of the experimenter. If this had been true, then one would have expected the subjects to have identified more than one overlapping triangle after the limited instruction. Since the data were consistent with Yakimanskaya's previously noted results, the experimenter feels that lack of understanding of directions is not sufficient explanation.

Second, the subjects seemed to employ search techniques (e. g.

left-to-right, circular); these techniques were more frequently employed by the older subjects. This result seemed to be clearer as the complexity of the figure increased, especially for Figure 1-5. Possibly, the behavior of the subjects reflected merely greater exposure to geometry instruction. The result is consistent, however, with the idea that perception of geometric figures is a developmental process.

No conclusion seems warranted with regard to the subjects' behavior in altering given diagrams to represent new conditions (i.e., in solving problems 1, 2, and 3). The lack of statistical relationship between problem solving ability and age was somewhat surprising. However, it does seem that if the development of search techniques were the result of increased exposure and instruction in geometry, then there might have been a corresponding increase in the number of correct solutions. The lack of relationship is interpreted as supporting the existence of stage-by-stage development of geometric perception, an interpretation which is consistent with the report of Wirszup (1971).

Finally, the limited instruction was somewhat effective. The inability of the subjects to transfer their learning (See Table 4) was disappointing, but not unreasonable. The instruction was designed to illustrate the existence of overlapping triangles rather than to provide a means whereby the subjects could identify new ones. The significance of the similarity of the distributions in Table 4 is not understood by the experimenter.

IMPLICATIONS

The consistency of the limited results of this study with those of Yakimanskaya and with the report of Wirszup suggest that much more work could be undertaken to determine the generalizability of the Soviet results. Such work might have a significant impact on the public school geometry curriculum in the U.S. In particular, instruction in geometry might become as coordinated across grade levels as the instruction on number systems currently is.

The report of Wirszup (1971) is a source of many hypotheses. The thrust of that report is that there are stages of development in acquiring skills in geometry, and the verification of these stages would seem to be an important prerequisite to improving the effectiveness of the geometry curriculum. Yakimanskaya (1958) stated that "there is a practical need to give pupils special exercises directed at developing their ability to change their perspective on a figure, taking note of ... the difficulty of isolating overlapping figures (p. 116)." In the sense that primary grade students do not seem able to identify overlapping figures, the same comment would apply to the sample of this study.

Finally, the relationship of geometric perceptual and problem solving skills to general problem solving skills would also seem to be a worthwhile direction of investigation. This experimenter has no guesses as to the nature of this relationship.

TABLE 1

Total Numbers of Identification of Overlapping and Non-Overlapping Figures

Source	Non-Overlapping	Overlapping
Figure 1-2		
Observed	30	0
Maximum Possible	30	15
Figure 1-4		
Observed	44	1
Maximum Possible	45	45
Figure 1-5		
Observed	59	1
Maximum Possible	60	60

TABLE 2

Distributions of Subjects According to Order of Identification of Non-Overlapping Triangels

Order of Identification	Age			
	7	8	9	10
Figure 1-2				
Upper First	1	4	2	1
Lower First	4	0	3	0
Figure 1-3				
Upper- First	2	1	2	1
Lower First	3	3	3	0
Figure 1-4				
Left to Right	2	2	1	0
Right to Left	3	2	2	1
Other	0	0	2	0
Figure 1-5[a]				
Clockwise	1	2	1	0
Counterclockwise	1	1	4	0
Other	2	1	0	0

[a]Two subjects are not included because they did not identify all four non-overlapping triangles.

TABLE 3

Distribution of Subjects According to

Response to Proble:. 1

Age	Correct Responses	Incorrect Responses
7	3	2
8	2	2
9	5	0
10	1	C

TABLE 4

Distributions of Subjects who Identified

Overlapping Triangles After Instruction

Figure	Age			
	7	8	9	10
1-5	4	2	4	1
1-4	2	1	2	1

APPENDIX A

Experimental Materials: Drawings and Problems

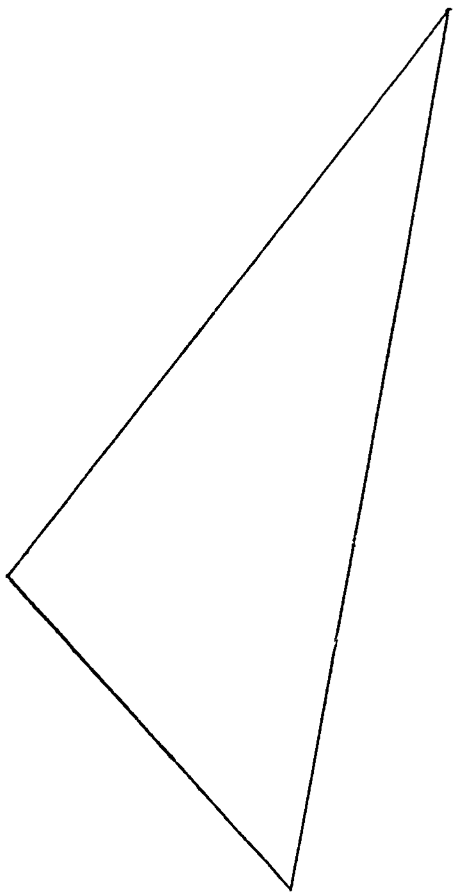

FIGURE I-1

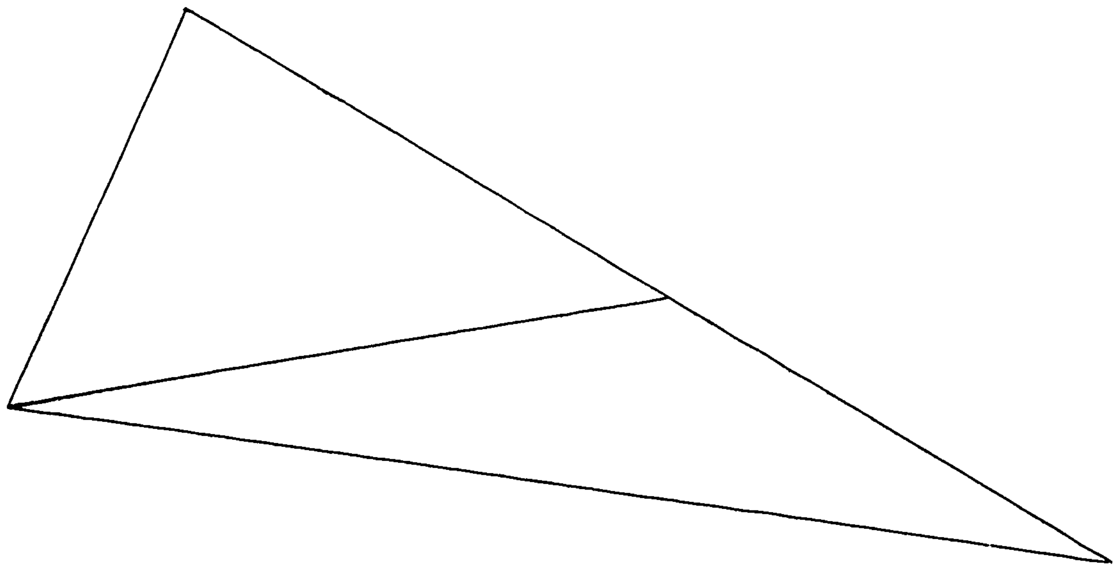

FIGURE I-2

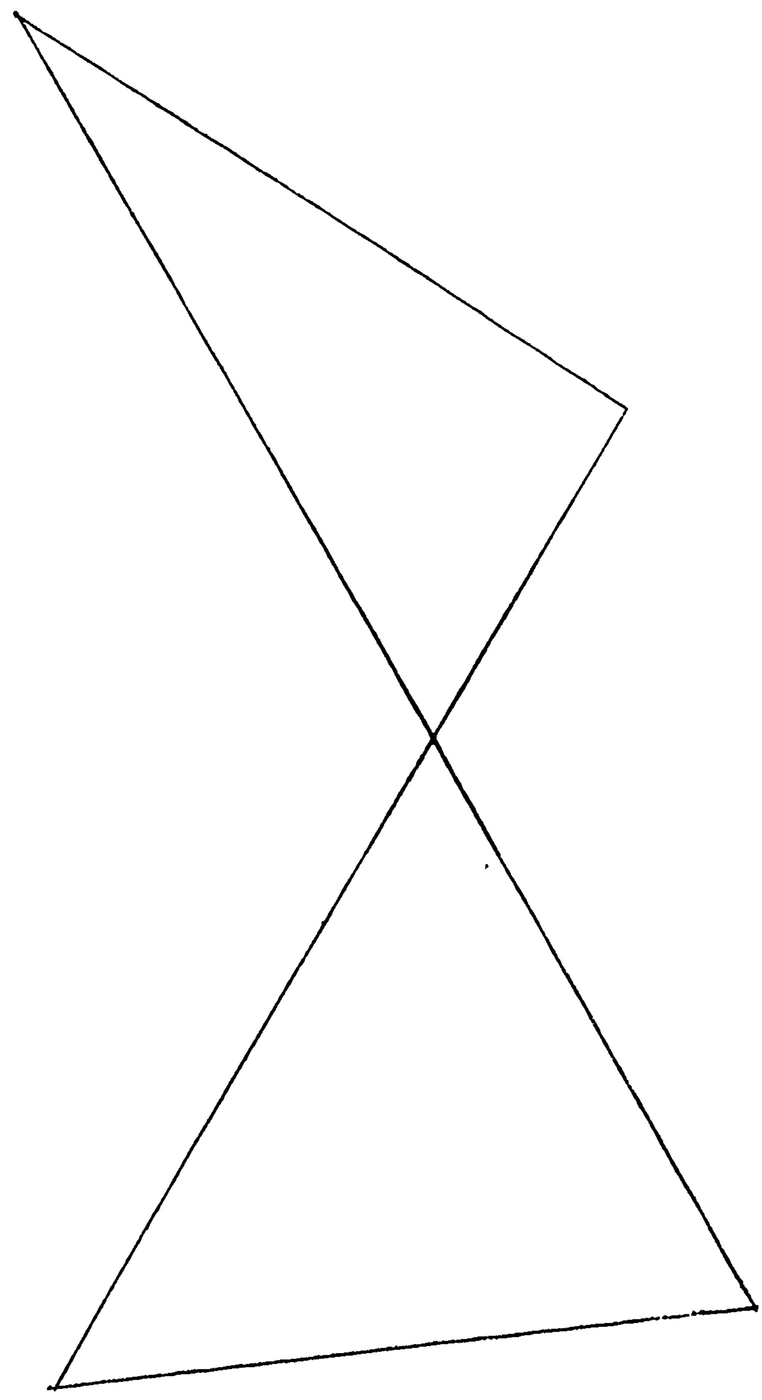

FIGURE I-3

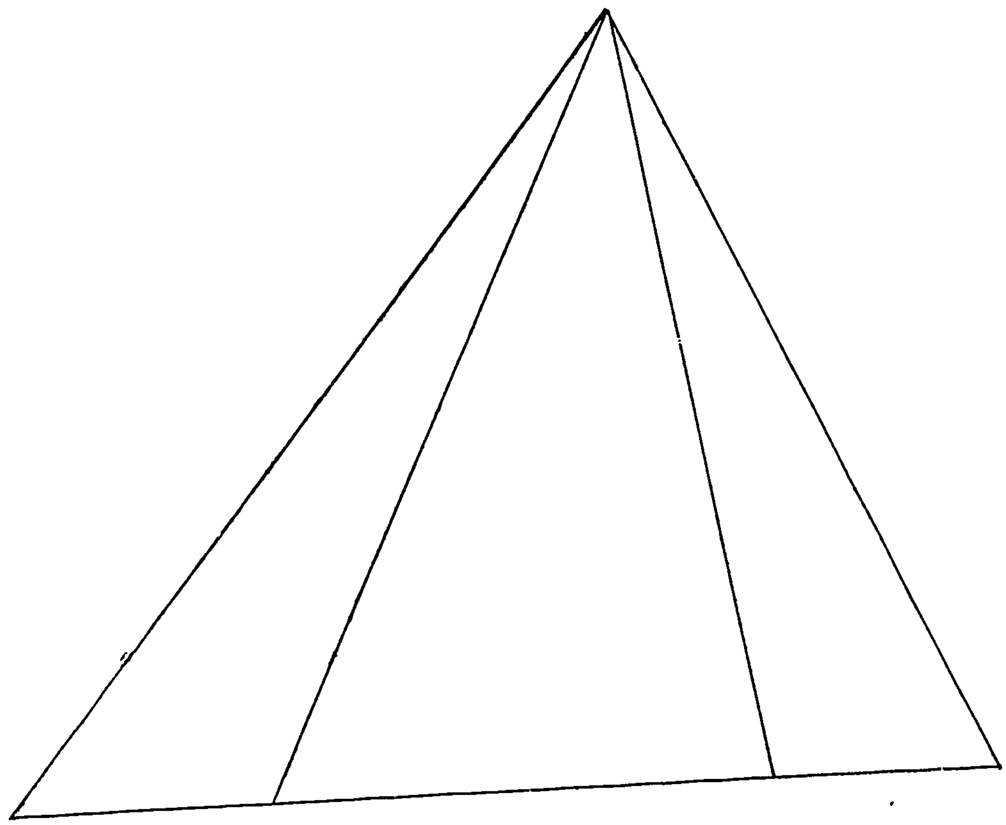

FIGURE I-4

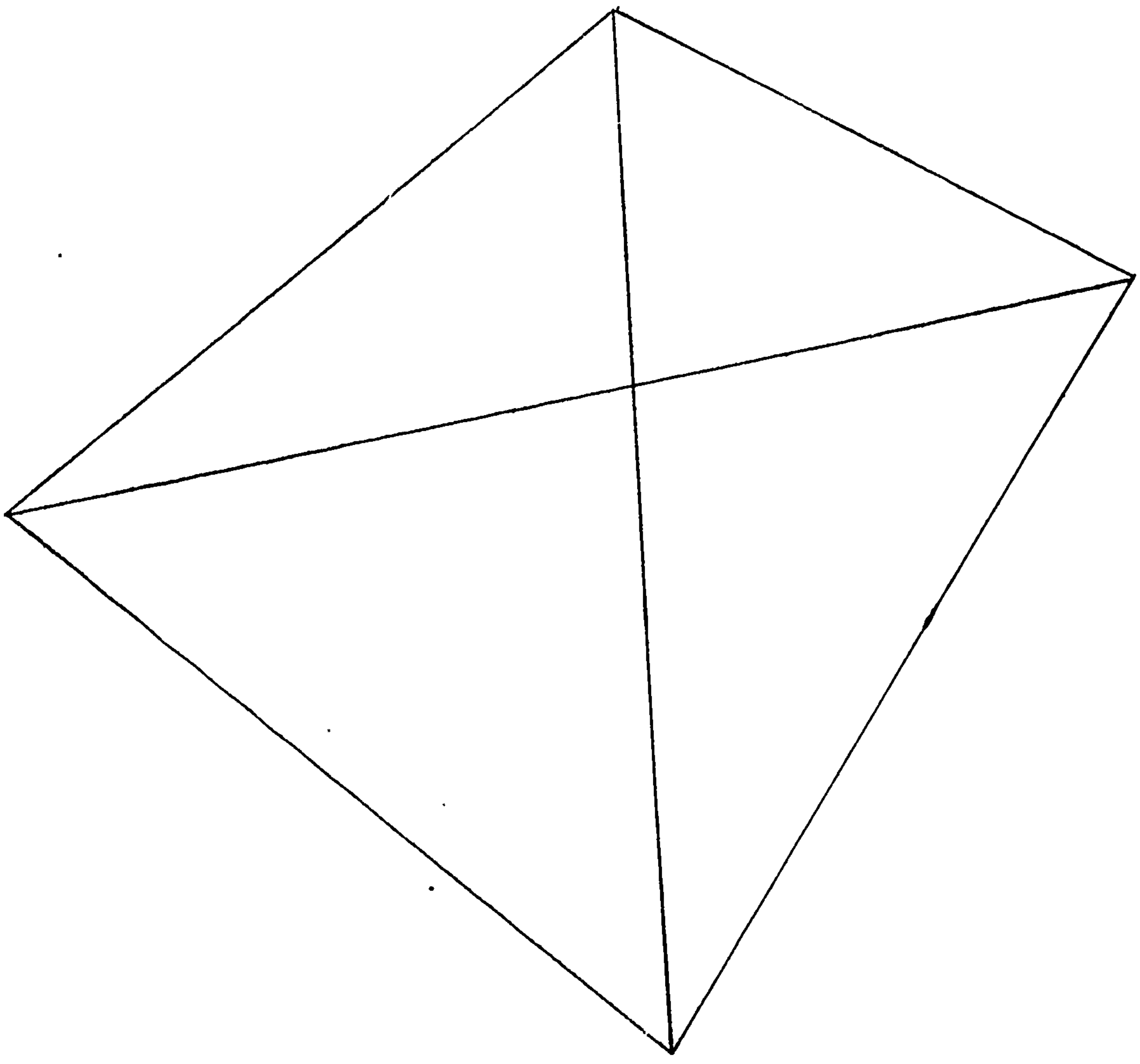

FIGURE I-5

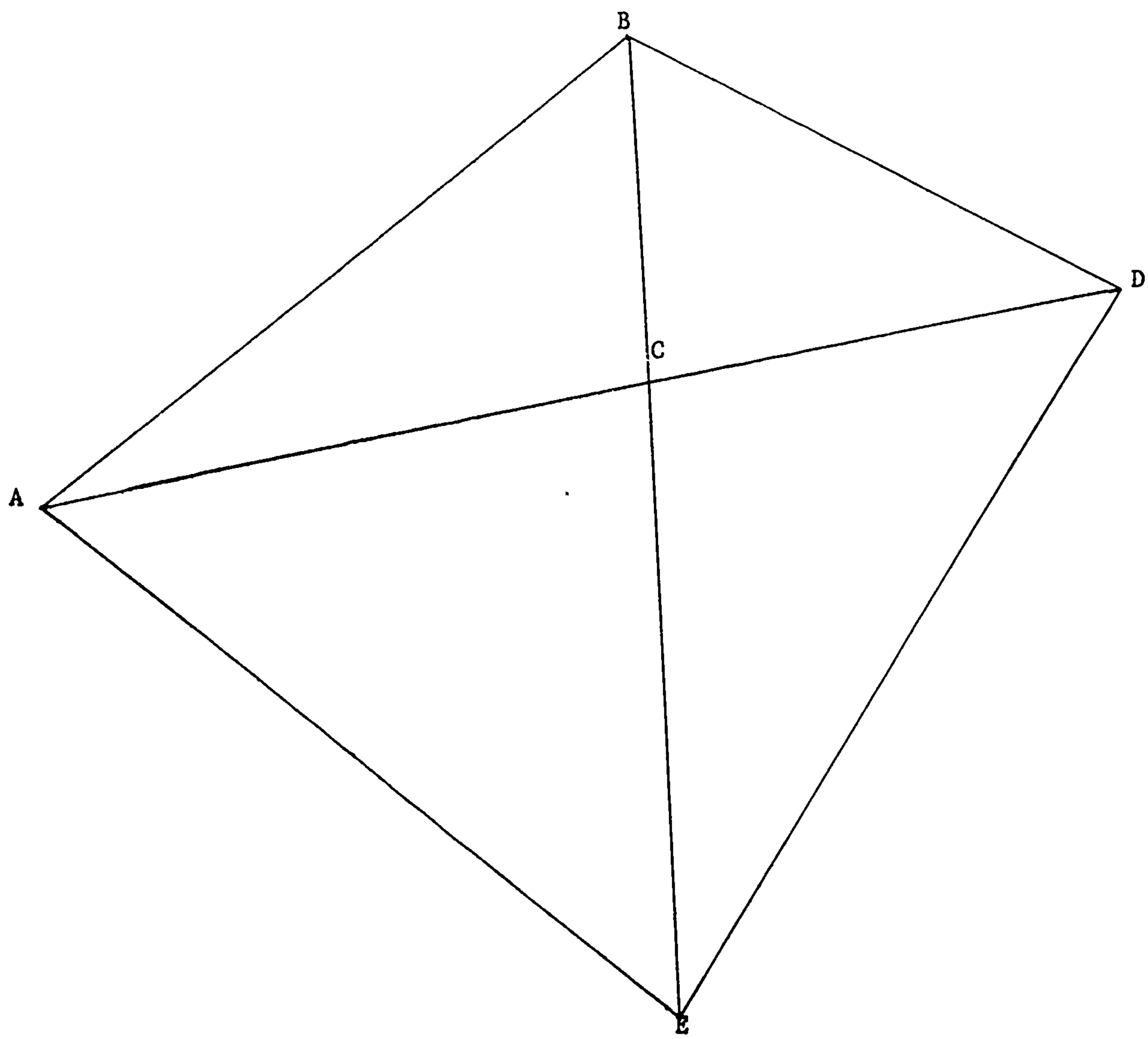

FIGURE I-5a

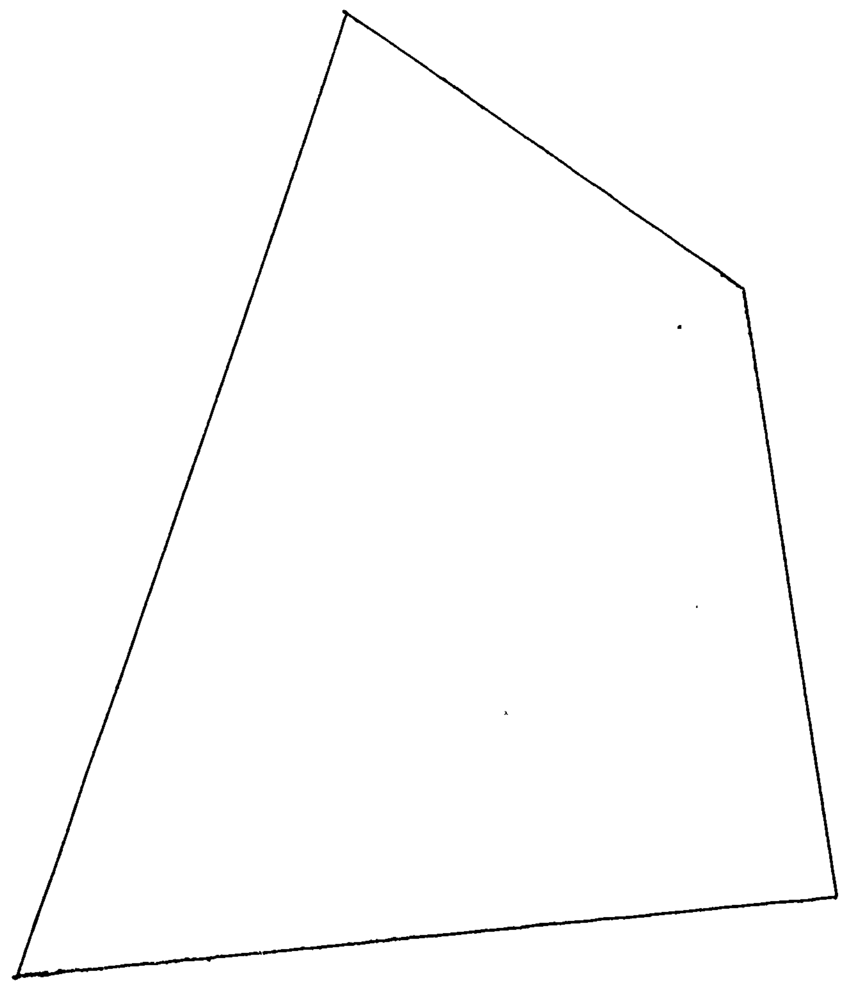

FIGURE II-1

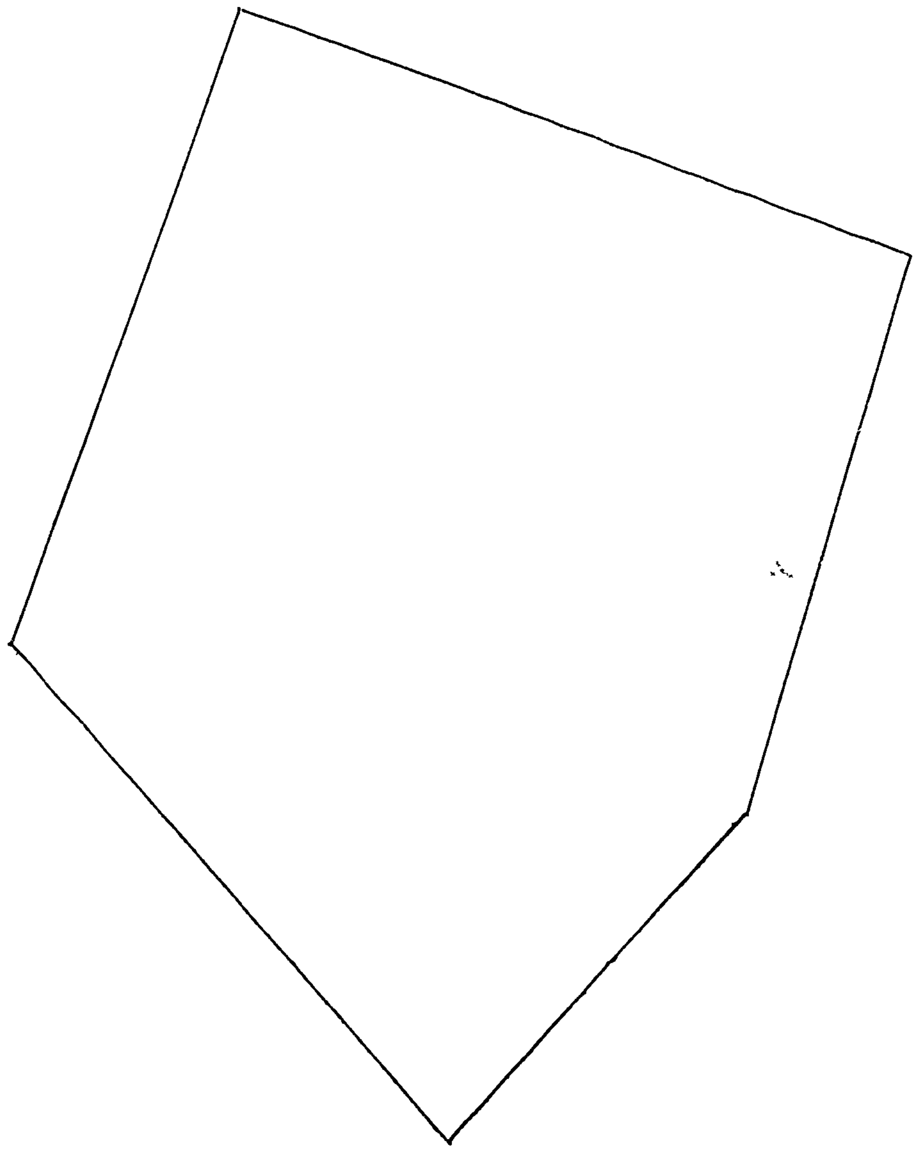

FIGURE II-2

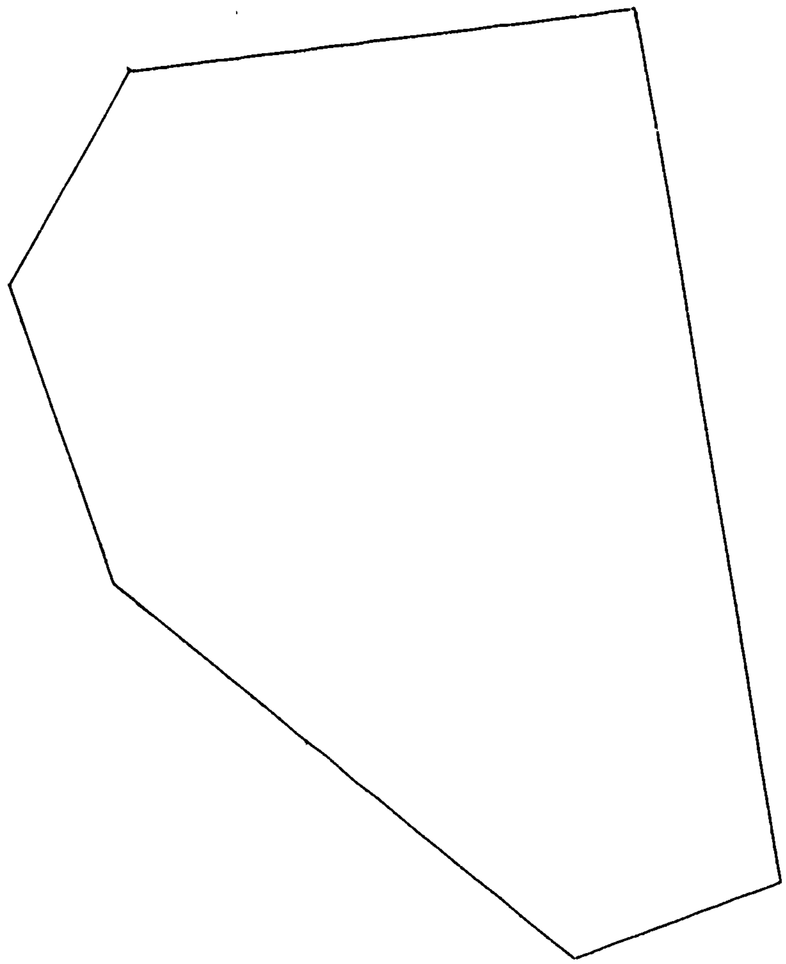

FIGURE II-3

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